# **Transformation toughening**

Part 5 Effect of temperature and alloy on fracture toughness

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The critical stress-intensity factor,  $K_c$ , of materials containing tetragonal  $ZrO_2$  was found to decrease with increasing temperature and  $CeO_2$  alloying additions, as predicted by theory. The temperature dependence of  $K_c$  was related to the temperature dependence of the chemical free-energy change associated with tetragonal—monoclinic transformation. Good agreement with thermodynamic data available for pure  $ZrO_2$  was obtained when the size of the transformation zone associated with the crack was equated to the size of the  $ZrO_2$  grains. The  $K_c$  against  $CeO_2$  addition data was used to estimate the tetragonal, monoclinic, cubic eutectoid temperature of  $270^\circ$  C in the  $ZrO_2$ -CeO<sub>2</sub> binary system.

# 1. Introduction

In Part 2 of this work [1] theory was presented showing that the contribution to fracture toughness,  $K_{c}$ , by a stress-induced transformation is proportional to the chemical free-energy change,  $|\Delta G^{c}|$ , associated with the transformation. For the  $ZrO_2$  (tetragonal)  $\rightarrow ZrO_2$  (monoclinic) transformation,  $|\Delta G^{c}|$  is known to decrease with increasing temperature and with alloying ZrO<sub>2</sub> with  $Y_2O_3$ , CeO<sub>2</sub> etc. In this part of the series of papers, experiments were designed to measure  $K_{c}$  as a function of temperature and alloy content. The temperature dependence of  $K_c$  was measured on polycrystalline  $ZrO_2$  and two-phase  $Al_2O_3 ZrO_2$  materials in which  $2 \mod \% Y_2O_3$  was alloyed with the  $ZrO_2$ . The fabrication conditions and general properties of these materials have been reported in Part 4 of this work [2].

A series of  $Al_2O_3$ -ZrO<sub>2</sub> materials in which CeO<sub>2</sub> was alloyed with the ZrO<sub>2</sub> phase was used to determine the effect of alloy content on  $K_e$ . As shown in Fig. 1, CeO<sub>2</sub> was a good candidate for this study since it forms an extensive solid-solution, tetragonal ZrO<sub>2</sub> phase-field and lowers the tetragonal  $\rightarrow$  monoclinic transformation temperature to less than  $25^{\circ}$  C at about 20 mol% CeO<sub>2</sub> addition.\* Initial ZrO<sub>2</sub>-CeO<sub>2</sub> sintering studies were not successful, i.e., higher CeO<sub>2</sub> contents (added to ZrO<sub>2</sub> powder as a soluble nitrate) resulted in a low-density material. Hot-pressing was avoided, since CeO<sub>2</sub> reduces to Ce<sub>2</sub>O<sub>3</sub> in environments produced by graphite dies. Attempts to sinter Al<sub>2</sub>O<sub>3</sub>-30 vol% ZrO<sub>2</sub> composite powders containing CeO<sub>2</sub> were successful in terms of density and phase content. Thus, these composite materials were chosen for the fracture toughness against alloying content studies.

# 2. Experimental procedure

# 2.1. Temperature dependence

Four materials were chosen for this study. Three of these were hot-pressed as detailed in Part 4 [2]:  $Al_2O_3-29.5 \text{ vol}\% \text{ Zr}O_2$  (plus  $2 \mod\% \text{ Y}_2O_3$ ),  $Al_2O_3-45 \text{ vol}\% \text{ Zr}O_2$  (plus  $2 \mod\% \text{ Y}_2O_3$ ) and  $ZrO_2$  (plus  $2 \mod\% \text{ Y}_2O_3$ ). The fourth material was an  $Al_2O_3-30 \text{ vol}\% \text{ Zr}O_2$  (plus  $2 \mod\% \text{ Y}_2O_3$ ) composite sintered to 97% of theoretical density in air for 1 h at 1600° C in which  $ZrO_2$  was retained in its tetragonal state. The composite powders were prepared for sintering by mixing

<sup>\*</sup>In contrast, the working tetragonal phase-field with  $Y_2O_3$  additions is limited to compositions between 2 and  $3 \mod \% Y_2O_3$  in both single-phase tetragonal  $ZrO_2$  [4] and  $Al_2O_3 - ZrO_2$  compositions [2].



Figure 1 A portion of the  $ZrO_2$ -CeO<sub>2</sub> phase diagram [3].

the required weight fractions of  $Al_2O_3$ ,\*  $ZrO_2^{\dagger}$ and yttrium nitrate<sup>‡</sup> by ball-milling in methanol ( $Al_2O_3$  balls and plastic bottle), drying, calcining at 500° C for 4 h, and isostatic pressing at 350 MPa. Small bar specimens cut from each material were polished in preparation for  $K_c$  measurements.

The indentation technique, developed by Evans and Charles [5], was used to measure  $K_c$  over the range of  $-196^{\circ}$  C (liquid nitrogen temperature) to 700° C. A Vickers diamond indentor mounted in tungsten carbide was used with a device which maintained a constant specimen temperature within the range noted. The device consisted of an internally-heated copper post, mounted within a metal flask. The flask was attached to an x-ystage used to translate the specimen relative to the indentor. The stage was mounted on top of a load cell and was insulated from the flask with a machinable ceramic. The specimen was spring-clip

<sup>11</sup>Added as Cerrium Nitrate, from Research Chemicals Corp.

loaded in a copper well attached to the post. A chromel-alumel thermocouple, spring-clip loaded to the external face of the specimen, was used to record temperatures. For the  $K_c$  measurements at temperatures less than 25°C, the flask was externally insulated and filled with liquid nitrogen. The nitrogen was allowed to slowly evaporate to achieve the desired specimen temperature. For measurements at higher temperatures, the flask was filled with insulating ceramic fibre, and the internal heater was controlled to achieve the desired temperature. Argon was forced into the metal flask to protect the copper parts and diamond from oxidation. Between measurements the indentor was held just above the specimen to avoid a large temperature differential when the indentor was again loaded into the specimen at 20 kg. A single specimen was used for the complete temperature range investigated; two measurements were made at each temperature.

Flexural strength measurements were made with one of the hot-pressed materials  $[(Al_2O_3-29.5 \text{ vol}\% \text{ ZrO}_2 (\text{plus } 2 \text{ mol}\% \text{ Y}_2O_3)]$  over the temperature range in which the  $K_c$  measurements were made. Bar specimens  $(3 \text{ mm} \times 6 \text{ mm} \times > 30 \text{ mm})$  were diamond-cut, ground and then annealed at 1300°C for 24 h to eliminate the surface compressive stresses developed due to the transformation of surface material during grinding [2]. Three strength measurements were made in liquid nitrogen, in a mixture of dry ice and methanol, at room temperature and in air at higher temperatures.

#### 2.3. Effect of alloying

As indicated above, a series of  $Al_2O_3$ -30 vol% ZrO<sub>2</sub> composite materials in which CeO<sub>2</sub> was incorporated were found suitable for fracture toughness against alloying content studies. Composite powders containing the appropriate weight fractions of  $Al_2O_3$ , ZrO<sub>2</sub><sup>¶</sup> and CeO<sub>2</sub><sup>∥</sup> were mixed and milled together in plastic bottles containing methanol and  $Al_2O_3$  mill balls, dried by flash evaporation, calcined at 500° C for 16 h, isostatically pressed into plates and sintered at 1600° C for 1 h. Sixteen compositions containing

<sup>\*</sup>Lindy B, Union Carbide Corp.

<sup>&</sup>lt;sup>†</sup>Zircar, Corp.

<sup>&</sup>lt;sup>‡</sup>Research Chemicals Corp.

<sup>&</sup>lt;sup>§</sup>From Lindy B, Union Carbide Corp.

<sup>&</sup>lt;sup>¶</sup> Sub-micrometre ZrO<sub>2</sub>, from Zircar Corp.

 $CeO_2$  contents of between 6 and 22 mol%  $CeO_2$ were fabricated. The densities of these composites ranged between 94% and 98% of theoretical density, based on the density of tetragonal ZrO<sub>2</sub> calculated using the lattice parameters a = 0.5126 nm and c = 0.5224 nm, as reported by Duwez and Odell [3]. X-ray diffraction analysis of the sintered surfaces showed that 100% of the  $ZrO_2$  was retained in its tetragonal structure for compositions containing  $\geq 12 \mod \%$  CeO<sub>2</sub>. Trace amounts of cubic ZrO<sub>2</sub> were observed for compositions containing 21 and 22 mol % CeO<sub>2</sub>, consistent with previous phase equilibria studies [3]. Increasing amounts of monoclinic ZrO<sub>2</sub> were observed as the CeO<sub>2</sub> content decreased from 11 to 6 mol %. Based on these observations, composites containing  $\geq 11 \mod \% \text{ CeO}_2$  were cut and polished for fracture toughness measurements at room temperature, as described above.

## 3. Results

#### 3.1. Temperature dependence

Fig. 2 illustrates that the fracture toughness decreases with increasing temperature for the four materials investigated. High, low and average values

of  $K_c$  are defined by the scatter bar at each temperature. These data were fit to a linear equation

$$K_{\rm c} = A - mT, \tag{1}$$

where T is temperature in degrees centigrade and the constants A and m are given in Table I.

The flexural strength data for the  $Al_2O_{3-}$ 29.3 vol% ZrO<sub>2</sub> (plus 2 mol% Y<sub>2</sub>O<sub>3</sub>) composites are shown in Fig. 3. These data show that strength decreases with increasing temperature. The dashed line illustrates the expected temperature behaviour of strength, based on the temperature behaviour of  $K_c$ , as reported in Table I for this composition and normalizing all data to the room-temperature value.

#### 3.2. Effect of alloying

Fig. 4 reports the  $K_c$  data against the CeO<sub>2</sub> addition to the ZrO<sub>2</sub> in the Al<sub>2</sub>O<sub>3</sub>-30 vol% ZrO<sub>2</sub> sintered materials. Data obtained for the composition containing 11 mol% CeO<sub>2</sub> is low due to its substantial (~30%) monoclinic-phase ZrO<sub>2</sub> content. Over the range where only the tetragonal ZrO<sub>2</sub>-phase is observed (12 to 20 mol% CeO<sub>2</sub>),  $K_c$  decreases with increasing CeO<sub>2</sub> content.  $K_c$ 



Figure 2 Critical stress-intensity factor plotted against temperature for the four materials investigated.

TABLE I Constants defining the temperature dependence of  $K_c$ 

Material	Fabriation conditions	<i>A</i> (MPa m <sup>1/2</sup> )	<i>m</i> (MPa m <sup>1/2</sup> ° C <sup>-1</sup> )	Correlation coefficient
$\frac{\text{Al}_2\text{O}_3-29.3 \text{ vol}\% \text{ ZrO}_2}{(\text{plus } 2 \text{ mol}\% \text{ Y}_2\text{O}_3)}$	Hot-pressed 1600° C/1 h	7.56	0.0044	0.95
$Al_2O_3 - 45 \text{ vol }\% \text{ Zr}O_2$ (plus 2 mol $\% \text{ Y}_2O_3$ )	Hot-pressed 1600° C/1 h	6.78	0.0029	0.92
$ZrO_2$ (plus 2 mol % $Y_2O_3$ )	Hot-pressed 1600° C/1 h	8.40	0.0041	0.99*
$Al_2O_3 - 30 \text{ vol }\% \text{ ZrO}_2$ (plus 2 mol $\% \text{ Y}_2O_3$ )	Sintered 1600° C/1 h	9.96	0.0054	0.97

\*Data at 100° C excluded.

appears to level-off to approximately  $6 \text{ MPa m}^{1/2}$ at the reported tetragonal-cubic phase boundary (compositions containing > 20 mol% CeO<sub>2</sub>). A linear relation was assumed over the range of 12 to 20 mol% CeO<sub>2</sub>, resulting in the relation

$$K_{\rm c} = 10.32 - 0.202M, \qquad (2)$$

where M is the mol % CeO<sub>2</sub> and  $K_c$  is in MPa m <sup>1/2</sup>.

# 4. Discussion

In Part 2 [1] of this series of papers, it was shown that the fracture toughness of a brittle material containing a phase which would undergo a stressinduced transformation could be expressed as

$$K_{\rm c} = \left[ K_0^2 + \frac{2V_{\rm i}E_{\rm c}R(|\Delta G^{\rm c}| - \Delta U_{\rm se}f)}{(1 - \nu_{\rm c}^2)} \right]^{1/2},$$
(3)

where  $K_0$  is the critical stress-intensity factor for the composite without the transformationtoughening phenomena,  $E_c$  and  $v_c$  are the elastic properties of the material,  $V_i$  is the volume fraction of the phase which could undergo the stressinduced transformation, R is the size of the



Figure 3 Flexural strength plotted against temperature for the  $Al_2O_3-29.3 \text{ vol}\%$  ZrO<sub>2</sub> (plus 2 mol% Y<sub>2</sub>O<sub>3</sub>) material. Specimens first annealed at 1300° C for 24 h.



Figure 4 Critical stress-intensity factor plotted against mol% CeO<sub>2</sub> at room temperature.

transformation zone adjacent to the crack and  $(|\Delta G^{c}| - \Delta U_{se}f)$  is the work-loss per unit volume during the stress-induced transformation. Since the magnitude of the chemical free-energy change associated with the transformation,  $|\Delta G^{c}|$ , is expected to exhibit the greatest dependence on temperature and alloying, relative to the other factors, it was predicted that the contribution of the stress-induced transformation to fracture toughness (i.e., the second term in Equation 3) would have the same temperature and alloy dependence as  $|\Delta G^{c}|$ . Based on the known temperature and alloying dependence of  $|\Delta G^{c}|$  for the  $ZrO_{2}(t) \rightarrow$  $ZrO_2(m)$  transformation,  $K_c$  is expected to decrease with increasing temperature and alloying content, which is the general result shown in Figs 2 and 4, respectively. The following paragraphs present a more detailed analysis and discussions of these data with reference to Equation 3.

## 4.1. Temperature dependence

Based on the assumption that  $|\Delta G^{c}|$  is the only temperature-dependent factor in Equation 3, data obtained during this study and reported in Part 4 [2] of this series of papers were used to calculate  $(|\Delta G^{c}| - \Delta U_{se}f)$  as a function of temperature for

comparison with the known temperature dependence of  $|\Delta G^{c}|$  for the transformation of pure  $ZrO_2$  [6]. This calculation started by determining the size of the transformation zone, R, for each material at room temperature, using the average room-temperature value of  $(|\Delta G^{c}| - \Delta U_{se}f)$  calculated in [2] for a series of  $Al_2O_3$ -ZrO<sub>2</sub> composites, values of  $K_0$  and  $E_c$  reported<sup>\*</sup> for each material in [2] and room-temperature  $K_{c}$  values reported here. Table II lists these values and the resulting value of R, as determined by rearranging Equation 3. It should be noted that in [1], it was hypothesized that  $R \simeq$  the grain size; calculated values of R shown in Table II are consistent with the grain sizes of the ZrO<sub>2</sub>-phase reported for the hotpressed materials in [2].

In the next step, values of  $K_c$  against temperature, reported in Table I, and the assumed temperature independent values of  $K_0$ ,  $E_c$ ,  $\nu_c$ ,  $V_i$  and R reported in Table II were used to calculate  $(\Delta G^c - \Delta U_{se}f)$  as a function of temperature for each material by rearranging Equation 3. These results are shown in Fig. 5,<sup>†</sup> Table II also reports the slope of each line  $[\delta(|\Delta G^c| - \Delta U_{se}f)/T]$  and the temperature where  $(|\Delta G^c| - \Delta U_{se}f) = 0$ ,  $(T_0)$ . The fifth line drawn in Fig. 5 is the temperature

<sup>\*</sup>As in [2],  $\nu_c$  was assumed to be 0.25.

<sup>&</sup>lt;sup>†</sup>The four lines coincide at 25° C since it was assumed in Step 1 that all materials had the same value of  $(|\Delta G^c| - \Delta U_{se} f)$  at 25° C.

T A B L E 11 Values used to analyse $K_{\mathbf{c}}$ against	t temperature da	ta							
Material	$K_{c}^{*}$ (MPa m <sup>1/2</sup> )	K <sub>0</sub> † (MPa m <sup>1/2</sup> )	$(\Delta G^{c} - \Delta U_{se}f)^{\dagger}$ (MJ m <sup>-3</sup> )	$E_{\mathbf{c}}^{\dagger}$ (GPa)	<sup>v</sup> c <sup>+</sup>	V <sub>i</sub>	R (µm)	$\frac{\delta( \Delta G^{c}  - \Delta U_{se}f)}{T}$ (MJm <sup>-30</sup> C <sup>-1</sup> )	T <sub>6</sub> (° C)
Al <sub>2</sub> $O_3$ -29.3 vol% ZrO <sub>2</sub> (plus 2 mol% Y <sub>2</sub> $O_3$ ) (hot-pressed)	7.45	4.10	1.88	333	0.25	0.293	66.0	-0.29	680
Al <sub>2</sub> $O_3$ -30 vol% ZrO <sub>2</sub> (plus 2 mol% $Y_2O_3$ ) (sintered)	8.83	4.10	188	333	0.25	0.30	1.53	0.27	730
$Al_2O_3 - 45$ vol % ZrO_2 (plus 2 mol % $Y_2O_3$ ) (hot-pressed)	8.30	3.70	1.88	285	0.25	0.45	1.10	-0.21	006
ZrO <sub>2</sub> (plus 2 mol% Y <sub>2</sub> O <sub>3</sub> ) (hot-pressed)	6.71	3.90	188	207	0.25	1.00	0.36	- 0.24	830
Average of four materials ZrO <sub>2</sub> (pure) [6] Zr <sub>0.96</sub> V <sub>0.96</sub> (ZrO <sub>2</sub> plus 2 mol % 0.5 O <sub>3</sub> ) [7,	8]							- 0.25 0.25	785 1200 600830
*Room-temperature values, Table I. †Room-temperature values, [2].									



Figure 5 Calculated values of  $(|\Delta G^{c}| - \Delta U_{se}f)$  plotted against temperature for four materials studied. Upper line is  $|\Delta G^{c}|$  against temperature for pure ZrO<sub>2</sub> as determined by Whitney [6].

dependence of  $|\Delta G^c|$  for pure  $ZrO_2$ , as previously reported by Whitney [6].

The calculations shown in Fig. 5 contain three results, which add greater confidence to the validity of the theoretical fracture mechanics calculations (Equation 3). First, since  $|\Delta G^c|$  is expected to exhibit the greatest temperature dependence relative to the other factors in Equation 3, the slopes of the four lines in Fig. 5 should be the same as  $\delta |\Delta G^{c}|/\delta T$  for  $Zr_{0.96}Y_{0.04}O_{1.98}(ZrO_{2})$ plus  $2 \mod \% Y_2O_3$ ). Although  $|\Delta G^c|$  against temperature data do not exist for this solid-solution compound, it is important to note that the slopes are nearly coincident (see Table II) with that of pure ZrO<sub>2</sub>, as reported by Whitney [6]. Second, the temperatures  $T_0$ , where  $(|\Delta G^c| - \Delta U_{se} f) = 0$ lie within the range of transformation temperatures (where  $\Delta G^{c} = 0$ ), see Table II, for  $Zr_{0.96}Y_{0.04}O_{1.98}$ powder [7, 8]. This result suggests that the residual strain energy associated with the ZrO<sub>2</sub> grains that contribute most to the fracture toughness is very small,  $\Delta U_{se} f \simeq 0$ . Third, the slope of the lines in Fig. 5 critically depend on the chosen value of R, i.e., larger or smaller values of R would not have resulted in the good agreement with  $\delta |\Delta G^c|/\delta T$  for pure ZrO<sub>2</sub>. Values of R calculated from roomtemperature data<sup>\*</sup> (Step 1) not only result in reasonable slopes for  $\delta |\Delta G^c|/\delta T$ , but they are also in good agreement with the size of the ZrO<sub>2</sub> grains as hypothesized by theory.

## 4.2. Effect of alloying

Based on the assumption that  $|\Delta G^{c}|$  is the only factor in Equation 3 affected by alloying CeO<sub>2</sub> with ZrO<sub>2</sub>, the  $K_{c}$  results presented in Equation 2 have been used to calculate the combined factor  $(|\Delta G^{c}| - \Delta U_{se}f)R$ . The linear expression resulting from combining Equations 2 and 3 with the appropriate values [2],  $K_{0} = 4.1$  MPa m<sup>1/2</sup>,  $E_{0} =$ 333 GPa,  $\nu_{c} = 0.25$  and  $V_{i} = 0.30$  for the Al<sub>2</sub>O<sub>3</sub>-30 vol% ZrO<sub>2</sub> (plus CeO<sub>2</sub>) compositions is (in MJ m<sup>-2</sup>)

$$(|\Delta G^{c}| - \Delta U_{se}f)R = 415 - 15.8M.$$
 (4)

Equation 4 can be used to estimate two thermodynamic properties of  $ZrO_2$ , which again

<sup>\*</sup>A second approach can also be used to determine R for each material by calculating the combined product  $R(|\Delta G^{c}| - \Delta U_{se}f)$  in Equation 3 as a function of temperature and assuming that  $\delta(|\Delta G^{c}| - \Delta U_{se}f)/\delta T = 0.248 \text{ MJ}$  m<sup>-3°</sup> C<sup>-1</sup> (the value of  $\delta |G_{c}|/\delta T$  for pure ZrO<sub>2</sub>) [6]. Using this approach, values of R for the four materials listed in Table II are 1.2, 1.65, 0.95 and 0.34  $\mu$ m, respectively.

adds greater confidence to the fracture mechanisms theory, as expressed in Equation 3.

First, by extrapolating the data obtained between M = 12 to  $20 \mod \%$  CeO<sub>2</sub> to M = 0, one obtains the value of  $(|\Delta G^{c}| - \Delta U_{se}f)R = 415 \text{ MJ}$  $m^{-2}$  for pure ZrO<sub>2</sub>. By choosing  $R = 1.5 \,\mu m$ , the value determined to be consistent with the  $K_{e}$ against temperature data for a similar, sintered  $Al_2O_3$ -30 vol% ZrO<sub>2</sub> composite discussed in the last section, one obtains  $(|\Delta G^{c}| - \Delta U_{se}f) =$  $275 \text{ MJ m}^{-3}$ . It is interesting to note that this value agrees almost exactly with the roomtemperature value of  $|\Delta G^{c}| = 290 \text{ MJ m}^{-3}$  for pure  $ZrO_2$  as previously calculated by Whitney [6] (see Fig. 5). Although this near-perfect agreement may be fortuitous, it again suggests that the residual strain energy,  $\Delta U_{se}f$ , associated with the transformed grains adjacent to the crack surfaces can be neglected in estimating their contribution to fracture toughness.

Second, previous phase equilibria work [5] in the  $ZrO_2$ -CeO<sub>2</sub> binary system has suggested that CeO<sub>2</sub>-additions in the range between 15 and  $20 \mod \% \text{ CeO}_2$  lowers the tetragonal  $\rightarrow \mod$ phase transformation temperature below 25° C.  $K_{c}$  measurements (see Fig. 2) clearly show that the tetragonal phase contributes to toughening over the complete range of  $CeO_2$ -contents studied (11) to  $22 \mod \%$ ). That is,  $K_c$  measurements strongly suggest that the eutectoid temperature is greater than 25°C. Using the fracture-mechanics data, one can estimate the eutectoid temperature by assuming that  $\Delta U_{se}f = 0$  and determining the value of M in Equation 4 where  $|\Delta G^{c}|R = 0$ . This condition exists when  $M = 26 \mod \%$  CeO<sub>2</sub>. By constructing a line between 1200° C and 26 mol% CeO<sub>2</sub> on the ZrO<sub>2</sub>-CeO<sub>2</sub> phase diagram and recognizing that the tetragonal + cubic phase-field exists when  $M = 20 \mod \% \text{ CeO}_2$ , one estimates the eutectoid temperature to be 270° C, which alters the phase diagram shown in Fig. 1 to that shown in Fig. 6.

## 5. Conclusions

(a) The fracture toughness of materials containing tetragonal  $ZrO_2$  decreased with increasing temperature and alloying addition, consistent with theoretical predictions.

(b) An analysis of the data suggests that the residual strain energy associated with the transformed  $ZrO_2$  grains can be neglected. Thus, the equation which appears to explain the contribution



Figure 6 A portion of the  $ZrO_2$ -CeO<sub>2</sub> phase diagram, in which the eutectoid temperature has been estimated through fracture-mechanics data.

of the stress-induced phase transformation to fracture toughness can be rewritten as

$$K_{\rm c} = \left[ K_0^2 + \frac{2|\Delta G^{\rm c}|E_{\rm c}V_{\rm i}R}{(1-\nu_c^2)} \right]^{1/2}.$$
 (5)

(c) The fracture-mechanics data, when analysed with respect to theory as expressed by Equation 3, best fitted the thermodynamic data for  $ZrO_2$  when it was assumed that the size of the transformation zone is the same size as the  $ZrO_2$  grains.

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